# QUANTIFIERS

Stanley Peters
Dag Westerståhl

## 1 Phenomena

This book is about the meaning of natural language expressions for quantification. It draws upon extensive research in linguistics and in logic by the authors and many other scholars, and it aims to weave these related but distinct threads together into a seamless book-length treatment of quantification.

The logical study of quantification is as old as logic itself, beginning in the work of Aristotle. This thread of research focuses on the meaning and inferential characteristics of quantifiers. Linguistic study of quantifiers originates with XXX, and until recently focused more on on the grammatical expression of quantification than on its meaning.

Linguists have borrowed heavily from logicians in semantically analyzing what quantifiers mean. Logicians found it easier to analyze the meaning of quantification over collections of discrete individuals than over parcels of non-discrete stuff (i.e. over what count nouns denote rather than the denotations of mass nouns). So the semantics of mass quantification has only recently begun to be developed (Link 1984, Landman 1989, Lønning 1987). Our exposition follows this historical order of development as well—treating quantifiers over discrete individuals first, and only later taking up quantification over collections and quantities of stuff. That seems to be the natural order of conceptual development.

The next section of this chapter provides examples illustrating the wide range of quantifiers expressed in natural languages and the variety of ways natural languages express them. The following section introduces differences in the semantic quantifier type, and distinguishes between explicitly quantifying expressions and those whose meaning only implicitly involves quantification. The section after that deals briefly with quantifier scope. The final section previews the historical tour in Chapter 2 through the logical analysis of quantifier meanings.

### 1.1 Some examples

Natural languages use quantifier expressions for talking about quantity of things or amount of stuff, such as dozens of eggs or liters of milk. These quantifier expressions include some of the ones Aristotle was concerned with, which are discussed further in Chapter 2.

| English | | Classical Greek | |
|---|---|---|---|
| count | mass | count | mass |
| *no* | *no* | | |
| *some* | *some* | | |
| *both* | | | |
| *few* | *little* | | |
| *a few* | *a little* | | |
| *several* | | | |
| *enough* | *enough* | | |
| *many* | *much* | | |
| *most* | *most* | | |
| *each* | | | |
| *every* | | | |
| *all* | *all* | | |

(1) Every dog barks.

(2) Susan likes most French films.

(3) Little water contains deuterium.

[GREEK EXAMPLES TOO]

These English and Classical Greek[CHECK THIS] expressions and modified forms like `(in)finitely many`, `too many`, `too few`, or `surprisingly few` appear in noun phrases as determiners of nominal expressions, or as part of such determiners.[1] In some languages the morphemes or words expressing these quantifiers do not occur in determiners of the nouns they quantify but rather as agreement markers on verbs or with various other grammatical functions [EXAMPLES].

Discontinuous parts of determiners like the following also express quantifiers.

| English | | Classical Greek | |
|---|---|---|---|
| count | mass | count | mass |
| *more... than* | *more... than* | | |
| *fewer... than* | *less... than* | | |
| *as many... as* | *as much... as* | | |

(4) More doctors than dentists are millionaires.

(5) As much sand as glass is silicon.

[1]We refer to the syntactic category of phrases like English `every Greek` as 'noun phrase' throughout this book for convenience. We are not aware of anything significant to the purposes of this book which hinges on whether these phrases are headed by nouns or by determiners as hypothesized by linguists who call them 'determiner phrases'.

Possessive determiners express quantifiers too.

| English | | Classical Greek | |
|---|---|---|---|
| count | mass | count | mass |
| *my* | *my* | | |
| *your* | *your* | | |
| *Pat's* | *Pat's* | | |
| *the chemistry professor's* | *the chemistry professor's* | | |
| ⋮ | ⋮ | ⋮ | ⋮ |

Quantifier expressions also include numerals, which have different syntactic properties from each group of expressions above.

| English | | Classical Greek | |
|---|---|---|---|
| count | mass | count | mass |
| *zero* | | | |
| *one* | | | |
| *two* | | | |
| *three* | | | |
| ⋮ | | ⋮ | |

Modified forms of numerals are also included,

| English | | Classical Greek | |
|---|---|---|---|
| count | mass | count | mass |
| *about five* | | | |
| *at least one* | | | |
| *at most six* | | | |
| *exactly three* | | | |
| *more than two* | | | |
| *fewer than four* | | | |
| ⋮ | | ⋮ | |

along with complex combinations like `between six and twelve`. In addition, proportional quantifiers exist for count and mass terms alike.

| English | | Classical Greek | |
|---|---|---|---|
| count | mass | count | mass |
| *half of the* | *half of the* | | |
| *at least a third of the* | *at least a third of the* | | |
| *at most two thirds of the* | *at most two thirds of the* | | |
| *more than half of the* | *more than half of the* | | |
| *less than three fifths of the* | *less than three fifths of the* | | |
| ⋮ | ⋮ | | |

Quantifiers are related to the indefinite and definite articles.

| English | | Classical Greek | |
|---|---|---|---|
| count | mass | count | mass |
| *a(n)* | | | |
| *the* | | | |

In addition, some adjectives express quantifiers.

| English | | Classical Greek | |
|---|---|---|---|
| count | mass | count | mass |
| *numerous* | | | |
| *innumerable* | | | |

So do some adverbs.

| English | | Classical Greek | |
|---|---|---|---|
| count | mass | count | mass |
| *always* | *always* | | |
| *mostly* | *mostly* | | |
| *mainly* | *mainly* | | |
| *frequently* | | | |
| *often* | *often* | | |
| *seldom* | *seldom* | | |
| *never* | *never* | | |

(6) Quadratic equations always have two solutions.

(7) Politicians are usually willing to help constituents.

(8) Men seldom make passes at girls who wear glasses.

Phrases like the following also express quantifiers.

| English | | Classical Greek | |
|---|---|---|---|
| count | mass | count | mass |
| *a couple of* | | | |
| *a lot of* | *a lot of* | | |
| *a small number of* | *a small amount of* | | |
| *a large number of* | *a large amount of* | | |
| *a finite number of* | *a finite amount of* | | |
| *an infinite number of* | *an infinite amount of* | | |
| *a odd number of* | | | |
| *a even number of* | | | |
| ⋮ | ⋮ | | |

Moreover, quantification may be expressed by non-phrasal constructions.

| English | | Classical Greek | |
|---|---|---|---|
| count | mass | count | mass |
| *there is/are* | *there is* | | |

## 1.2 Semantic Types of Quantification in Natural Languages

Natural language quantifiers can be classifed according to their semantic type in addition to their syntactic expression. Quantificational morphemes, words, and phrases of natural languages vary in the semantic type of quantifier expressed. When the differences adequately recognized, no particular problem exists in analyzing what each quantifier means.

The quantifiers exemplified in the first table of Section 1.1 are semantically of type $\langle 1, 1 \rangle$. Roughly speaking this means they are relations between two sets of things or stuff. In other words, a quantified sentence like (9)

(9) Every dog is barking.

states that $every(D, B)$ holds true, where $D$ is the set of dogs, $B$ is the set of things that are barking, and *every* is a relation between sets. The type $\langle 1, 1 \rangle$ quantifier *every* is a particularly simple relation to describe; it is just the subset relation $\subseteq$.

The next chapter explains more precisely what this amounts to. By contrast the English noun phrases `everything`, `something`, `nothing`, `everyone`, `someone`, `no one` and the Classical Greek noun phrases `GRRR` express type $\langle 1 \rangle$ quantifiers, [KEEP THE FOLLOWING?] as do the non-phrasal constructions in the last table above. Roughly speaking they are properties of sets of things or stuff. The discontinuous determiners `more...than` and `as many...as` in the second table of Section 1.1 express type $\langle 1, 1, 1 \rangle$ quantifiers. Roughly speaking these are relations between three sets of things or stuff.

Thus statements (10)–(12)

(10) Something is a success.

(11) Many people are millionaires.

(12) More doctors than dentists are millionaires.

describe a model with domain $M$ as satisfying (10′)–(12′)

(10′) $\exists_M(A)$

(11′) $many_M(B, C)$

(12′) $more\text{-}than_M(D, E, C)$

respectively, where $\exists_M$, $many_M$, and $more\text{-}than_M$ are generalized quantifiers as described more specifically in Chapter 2, and $A$ is the set of things that are a success, $B$ the set of people, $C$ the set of millionaires, $D$ the set of doctors and $E$ the set of dentists (according to the model $\mathcal{M}$).

### 1.2.1 Explicit Quantifiers vs. Implicit Quantification

It is useful to distinguish natural language phrases like the ones discussed so far, which express quantification over explicitly mentioned things or stuff, from other expressions whose meaning implicitly involves quantification. The latter include a wide range of items that nowadays are commonly analyzed in terms of quantification. For instance, the past tense may be analyzed as existential quantification over times preceding the present. The deontic modal `must` may be analyzed as universal quantification over possible worlds in which all obligations are complied with. The attitude verb `believe` may be analyzed in terms of universal quantification over possible worlds in which all a subject's beliefs are true. And so on.

These implicitly quantificational expressions have in common that sentences containing them typically do not also contain an expression that explicitly denotes the entities over which these expressions implicitly quantify. There need not be any time-denoting expression in a past tense sentence.

(13) Mary was at home.

Modal and belief statements do not contain expressions denoting possible worlds.

(14) John must be on time.

(15) I believe you are skeptical.

In terming such quantification 'implicit', we are not advocating an analysis in terms of times or possible worlds.[2] Rather we mean simply to point out that one who chooses to employ quantification in the semantic analysis of these expressions will quantify over entities that are not explicitly mentioned. These cases contrast sharply with the explicitly quantificational statements (10)–(12).

Borderline cases exist, and have been puzzlingly problematic. For instance, the truth conditions of sentence (16)

(16) The men slept.

are close if not identical to those of (17).

(17) Each man slept.

However, this does not necessarily show that the definite article `the` in (16) is a quantifier. Sentence (16) might acquire these truth conditions by a very different route than (17) does. Sentence (18)

(18) No man didn't sleep.

clearly acquires the same truth conditions as (17) by a different route. While (17) and (18) are logically equivalent, they are not alike in all respects; (18) contains more 'logical operators' than (17) does, for example. By the same token,

[2] Nor in terms of such alternatives as events, situations, or sets thereof.

(16) could be analyzed as containing one set-denoting term (the plural noun phrase `the men`) and distributing a collective predication over the members of that set. Thus (16) would have the same truth conditions as (17) and (18) even if it did not contain any item that explicitly expresses quantification. We return in later chapters to the characterization of implicit quantification. For now, suffice it to observe that we should resist temptation to call something a quantifier expression simply on the grounds that its meaning can be analyzed in terms of quantification, and inquire further whether the things it would quantify over are explicitly mentioned.

## 1.3 Quantifier Scope

### 1.3.1 Monadic Quantifiers

Natural language expressions for type $\langle 1 \rangle$ quantifiers such as `everyone` and `most French films` need one additional ingredient to yield a statement. This ingredient is commonly called the scope of the quantifier expression. Unlike quantifiers do in logical languages, the scope of a natural language quantifier may not be uniquely determined by the syntactic structure a quantifier appears in.

(19) Bill admitted Monica gave him many presents.

(a) He made many admissions that she gave him a present.

(b) He admitted (perhaps only once) that she gave him many presents.

Ambiguities resulting from lack of explicitly demarcated scope may be one indicator of explicit quantifiers.

Note that sentence (20)

(20) The novices chose a mentor.

which is similar in structure to (16), unambiguously entails that all novices have the same mentor. Likewise sentence (21) is unambiguous and entails nothing about whether or not the mentors are all different.

(21) The novices chose mentors.

In striking contrast, sentence (22), which contains both an explicit quantifier and the indefinite article exhibits quantifier scope ambiguity.

(22) Every novice chose a mentor.

Just such facts constitute reasons for questioning the hypothesis that the definite plural noun phrases in (16), (20) and (21) are universal quantifiers, and suspecting that universal quantification instead figures in the analysis of these sentences' meaning only in explaining how the predication expressed by the verb distributes over members of the collection that the noun phrase denotes.

### 1.3.2 Polyadic Quantifiers

The quantifiers discussed so far all 'bind one variable' in their scope. The notion of generalized quantifier, however, allows for the possibility of quantifiers that bind two or more variables in their scope. Do such quantifiers occur in natural languages?

They do indeed seem to exist. One example is the reciprocal expression `each other` in English, as in sentence (23).

(23) Congressmen must refer to each other indirectly.

Clearly quantification is called for in of semantically analyzing (23), whose meaning is something like: 'each congressman must refer to every other congressman indirectly'. But does `each other` involve quantification only implicitly, like the past tense, the modal `must` and the verb `believe`? Evidently not, as the scope of the polyadic quantifier `each other` is not uniquely determined by its position in syntactic structure. For example, in sentence (24),

(24) John and Bill think they are taller than each other.

the scope of the reciprocal can either be the subordinate clause which is the complement of `think`, giving the silly interpretation

(24 a) Each thinks: We're taller than each other.

or the reciprocal's scope can be the entire sentence, which gives the obviously sensible interpretation in this case.

(24 b) Each thinks: I'm taller than him.

We analyze the very interesting type $\langle 1, 2\rangle$ quantifier expressed by reciprocal phrases like `each other` in more detail in Chapter 000. For now, we note simply that the following examples show that its meaning does not always decompose into two universal quantifications.

(25) a. Five Boston pitchers sat alongside each other.

b. The pirates stared at each other in surprise.

c. They stacked tables on top of each other to reach a high window.